# 从对趋肤效应的解释有疑谈起


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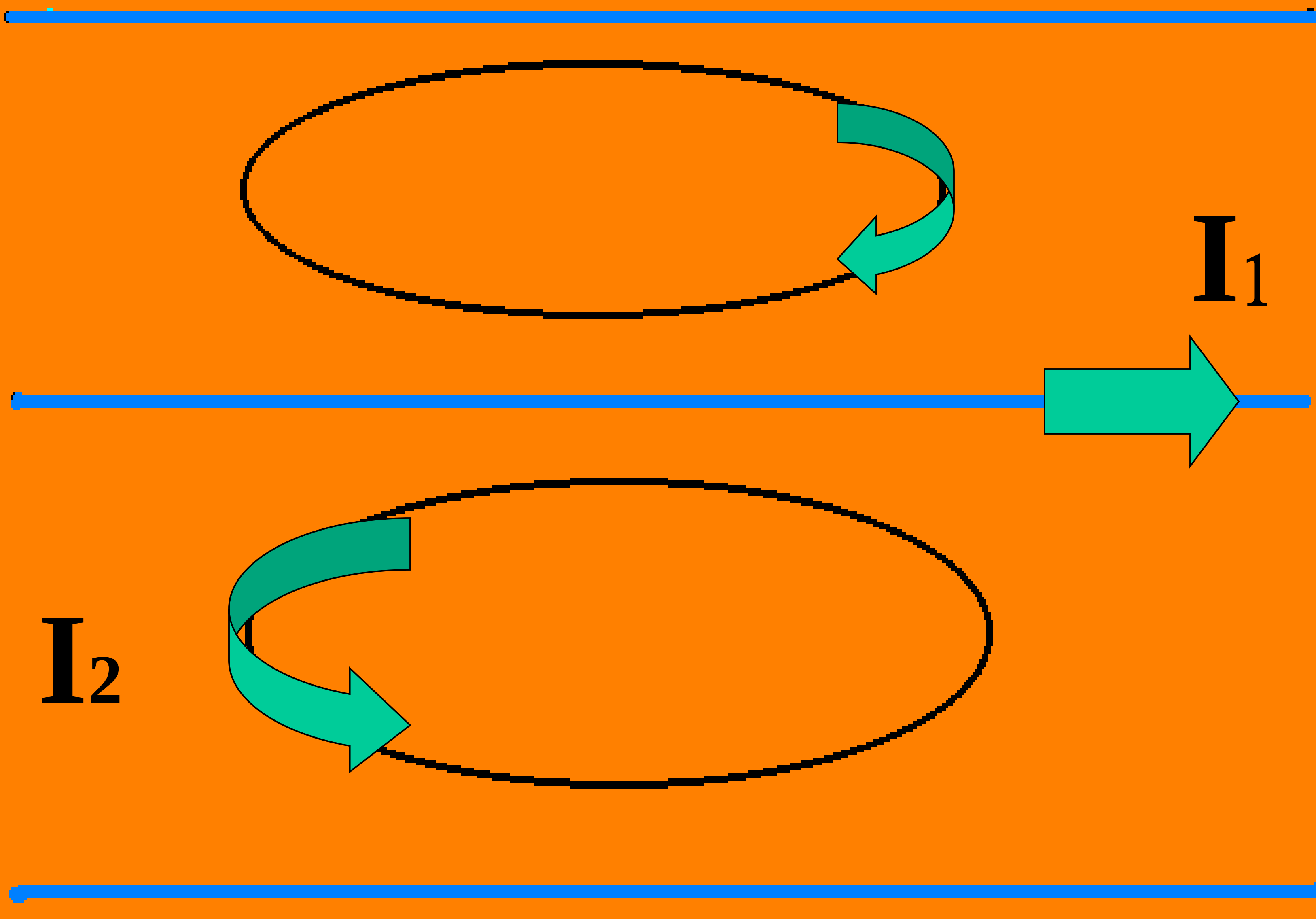
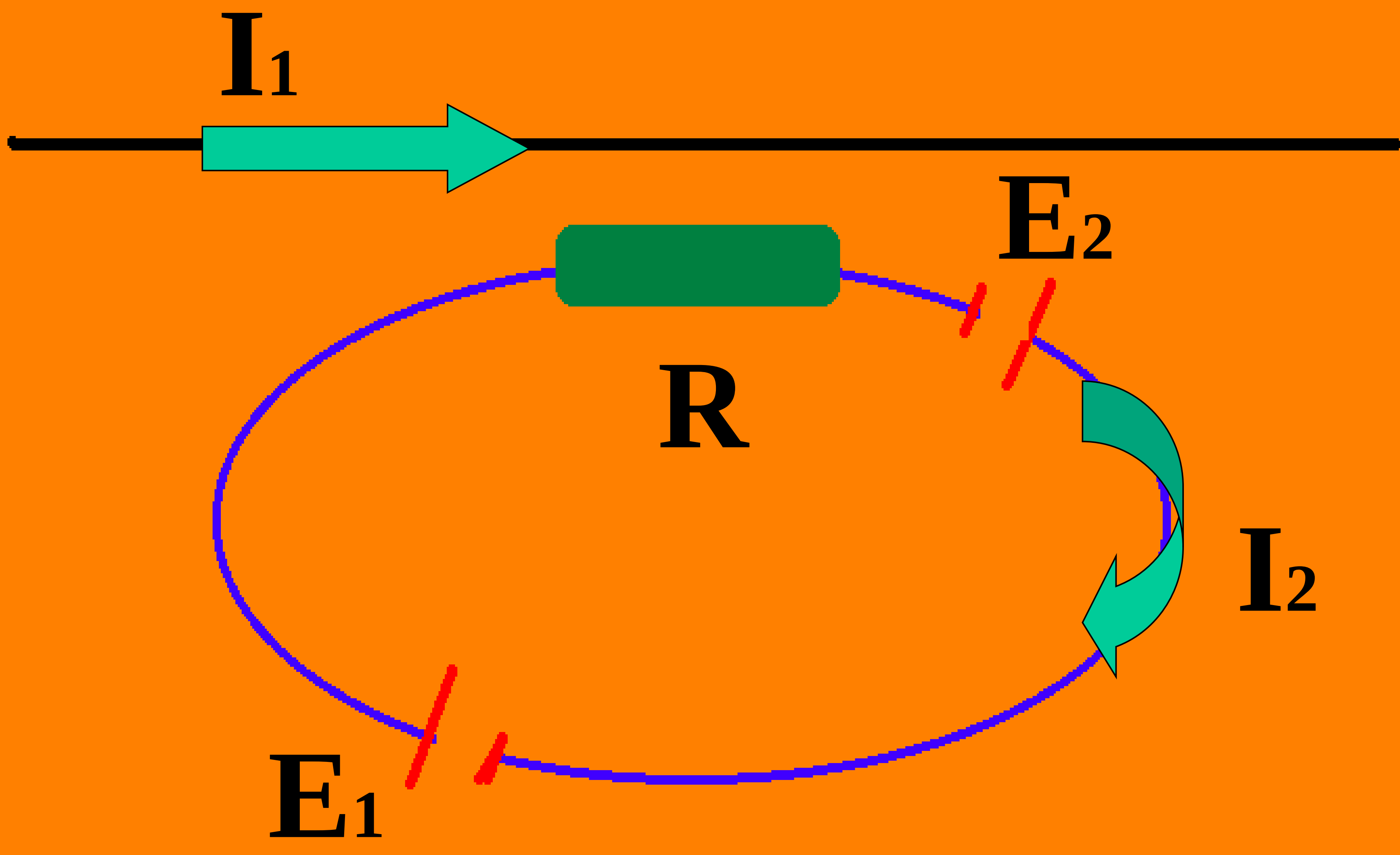
# 关键词：

趋肤效应，麦克斯维方程，自感，互感，贝塞尔函数



$I_1$
$I_2$



I1
E2
R
I2
E1

$$\varepsilon_1 = -M\frac{dI_1}{dt}, \varepsilon_2 = -L\frac{dI_2}{dt} \qquad (1)$$

$$-L\frac{dI_2}{dt} - M\frac{dI_1}{dt} = I_2 R \qquad (2)$$

$$\frac{dI_2}{dt} + \frac{R}{L} = -\frac{MI_0\omega}{L}\cos\omega t \qquad (3)$$

$$I_2 = X(t)e^{-\frac{R}{L}t}$$

其中 $X(t) = \int Ke^{\frac{R}{L}t} \cos \omega t dt$

$$= \frac{Ke^{\frac{R}{L}t}}{\sqrt{\left(\frac{R}{L}\right)^2 + \omega^2}} \sin(\omega t + \varphi) + C$$

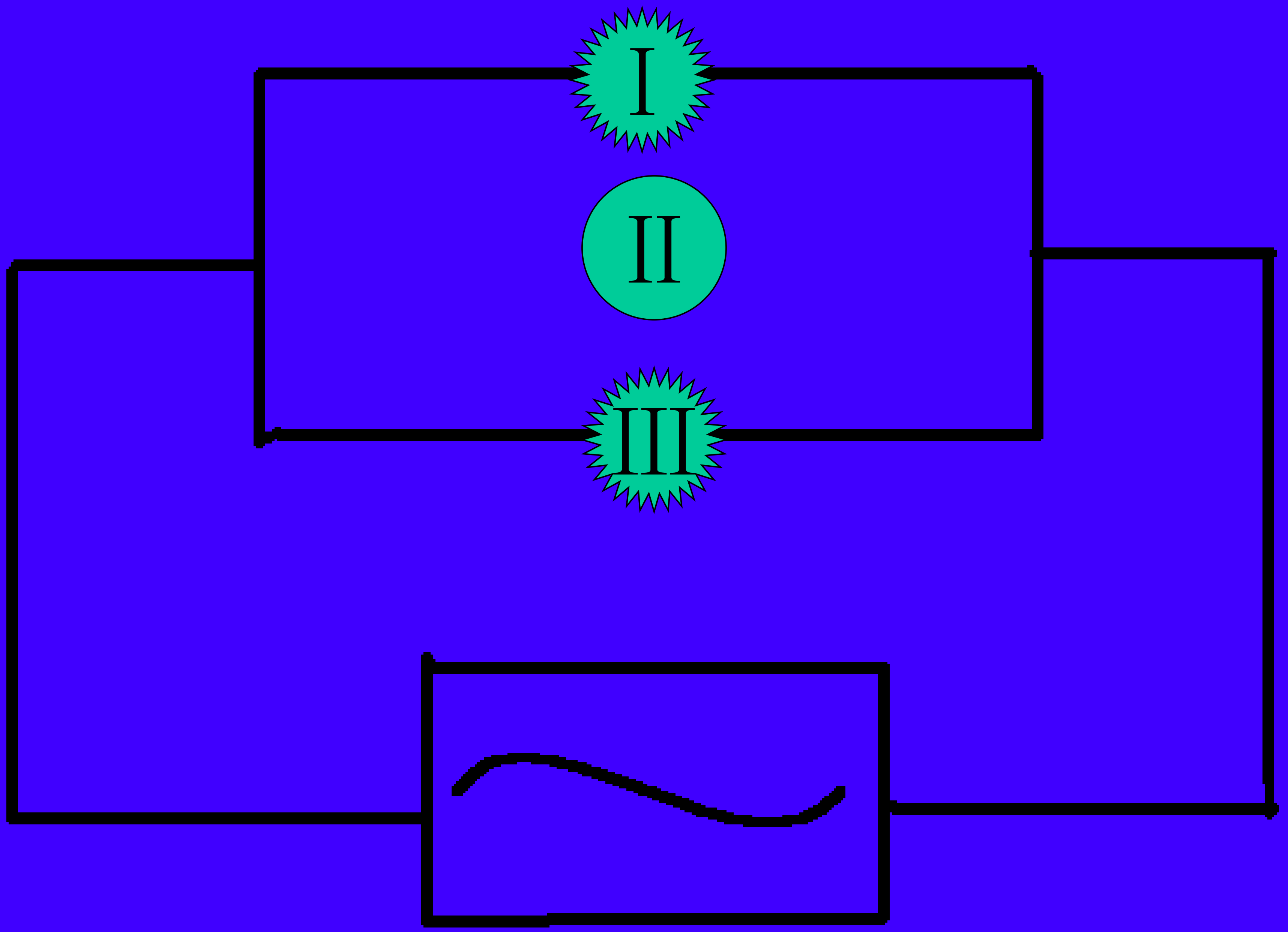
$$I_2(t) = -\frac{MI_0\omega}{L\sqrt{\left(\frac{R}{L}\right)^2 + \omega^2}}\sin(\omega t + \varphi)$$

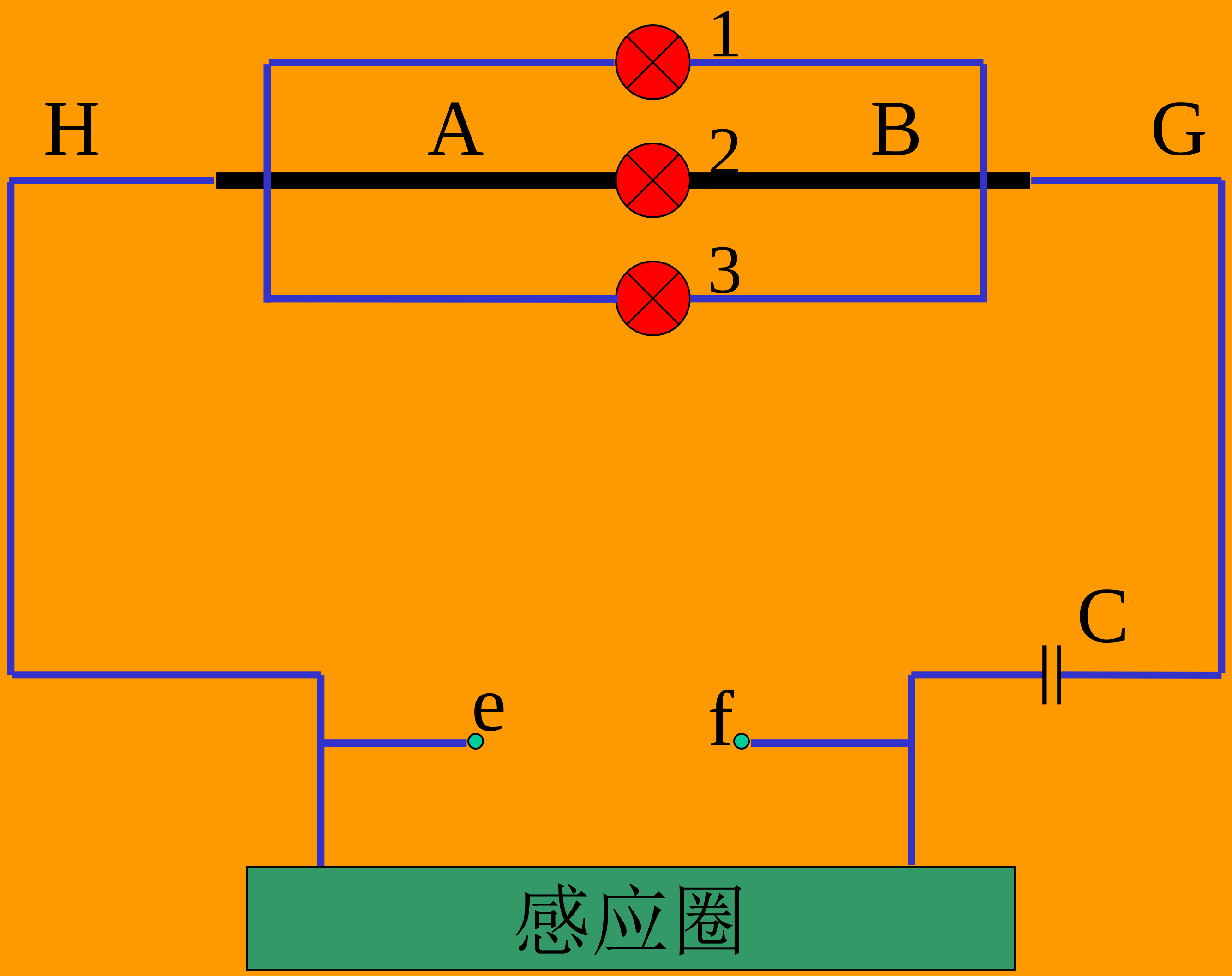
$$= \frac{MI_0\omega}{\sqrt{R^2 + L^2\omega^2}}\sin(\omega t + \varphi + \pi)$$

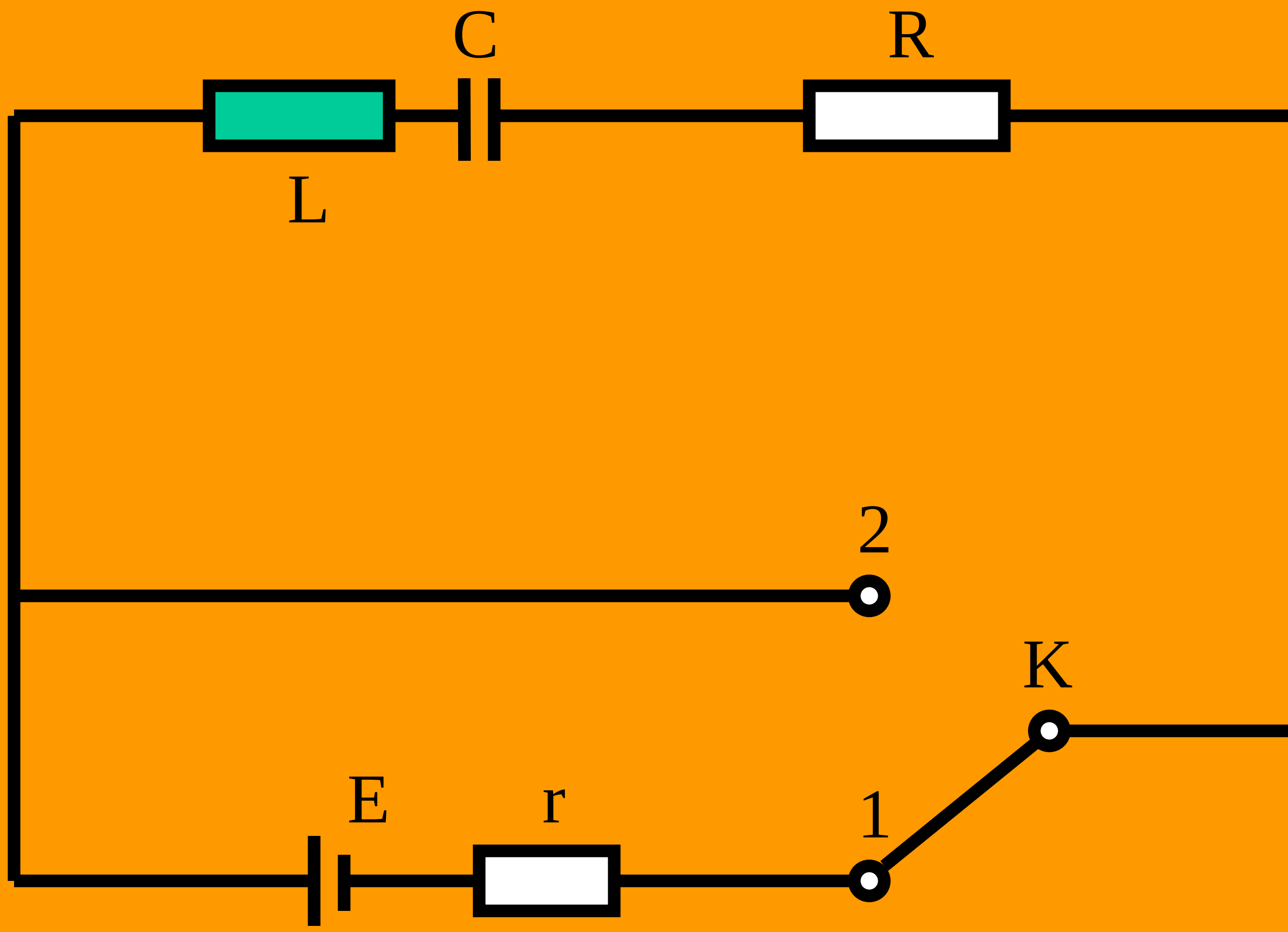
若 $R << \omega L$,

则：$\varphi \to 0$，

$$\therefore I_2 \approx \frac{MI_0\omega}{\sqrt{R^2 + L^2\omega^2}} \sin(\omega t + \pi)$$


I
II
III


1
H
A
2
B
G
3
C
e
f
感应圈


C
R
L
2
K
E
r
1

振荡频率

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

**Maxwell方程：**

$$\begin{cases} \nabla \times \vec{\dot{H}} = \vec{\dot{J}} = \sigma \vec{\dot{E}} \\ \nabla \times \vec{\dot{E}} = j\omega\mu \vec{\dot{H}} \end{cases}$$

在柱坐标中可化为：

$$\begin{cases} \dfrac{1}{r}\dfrac{d}{dr}\left(r\overrightarrow{\dot{H}}\right) = \sigma\overrightarrow{\dot{E}} \\[2em] \dfrac{1}{r}\dfrac{d}{dr}\left(r\overrightarrow{\dot{E}}\right) = j\omega\mu\overrightarrow{\dot{H}} \end{cases}$$

进一步化简为：

$$\frac{d^2 \vec{\dot{E}}}{d^2 (qr)^2} + \frac{1}{qr}\frac{d\vec{\dot{E}}}{d(qr)} + \vec{\dot{E}} = 0$$

$$\frac{d^2 \vec{\dot{H}}}{d(qr)^2} + \frac{1}{qr}\frac{d\vec{\dot{H}}}{d(qr)} + \left(1 - \frac{1}{q^2 r^2}\right)\vec{\dot{H}} = 0$$

其中 $q = \sqrt{-i\omega\mu\sigma}$

若令 $k = \sqrt{\omega\mu\sigma}$ ，则 $qr = \sqrt{-i} \cdot kr$ 。

解出：

$$\dot{E} = \dot{C} \cdot J_0(qr)$$

由前及递推公式 $J_0'(x) = -J_1(x)$ 知：

$$\dot{H} = -\frac{\sigma \dot{C}}{q} \cdot J_1(qr)$$

$$\oint_L \vec{H} \cdot d\vec{l} = \sum I$$

当r=a时，则可推出：

$$\begin{cases} \dot{C} = \dfrac{q\dot{I}}{2\pi a\sigma J_1(qa)} \\ \dot{E} = \dfrac{q\dot{I}}{2\pi a\sigma} \cdot \dfrac{J_0(qr)}{J_1(qa)} \\ \dot{H} = \dfrac{I}{2\pi a} \cdot \dfrac{J_1(qr)}{J_1(qa)} \end{cases}$$

$$\dot{j} = \sigma\dot{E} = \frac{q\dot{I}}{2\pi a} \cdot \frac{J_0(qr)}{J_1(qa)}$$

$$J_0(qr) = J_0\left(kr\sqrt{-i}\right)$$

$$= ber_0(kr) + i\,bei_0(kr)$$

$$J_1(qa) = J_1\left(ka\sqrt{-i}\right)$$

$$= ber_1(ka) + i\,bei_1(ka)$$

# 谢谢！